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Free convection above a uniformly heated horizontal circular disk

J. H. MERKIN
Department of Applied Mathematical Studies, University of Leeds, Leeds LS2 9JT, U.K.

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Abstract—The free convection boundary layer above a uniformly heated horizontal disk is considered. The equations of motion are solved numerically and by a series expansion valid near the circumference. It is shown that near the edge of the disk the flow is basically the same as on a plate, with the importance of the curvature effects increasing as the centre is approached. It is also shown that near the centre, the boundary-layer thickness increases very rapidly and that the solution splits up into two distinct regions. There is a thin viscous region next to the disk of thickness $O(r^{2/3})$, where r measures distance from the centre, and a very much thicker inviscid region of thickness $O(r^{-2/3})$.

1. INTRODUCTION

IN THIS paper we consider the free convection boundary-layer flow above a uniformly heated horizontal circular disk. The related problem in which the temperature of the disk is some prescribed constant value above ambient (as opposed to the present case where a constant heat flux is prescribed on the disk) has been treated by Zakerullah and Ackroyd [1] and Merkin [2]. We follow the same solution procedure as in refs. [1] and [2], obtaining first a series solution valid near the circumference of the disk, then extending this series by a numerical solution of the full boundary-layer equations. The numerical integration can proceed almost to the centre of the disk and we then complete the flow description by discussing the nature of the solution close to the centre.

The solution at the circumference is the same as on a plate, the curvature effects being a perturbation on this solution. However, as the centre of the disk is approached, these curvature effects become more important and the flow divides up into two regions. This is clearly shown by the numerical solution. There is a thin inner viscous region of thickness of $O(r^{2/3})$ (where r is a (non-dimensional) coordinate measuring distance from the centre) in which the temperature is almost constant, and the pressure is large (and negative) and almost uniform, of $O(r^{-2/3})$. Outside this region is a thick outer inviscid region, of thickness $O(r^{-2/3})$, which is driven by the pressure gradient induced by the inner region. This outer region is in some way similar to the eruption region near the top of a heated sphere [3, 4] in which the boundary layer leaves the surface of the disk to form a buoyant plume above it, though here the basic structure is somewhat different. In this case the boundary layer leaves the surface as a consequence of the flow converging onto the centre of the disk, manifesting itself as a singularity in the axi-symmetric boundary-layer equations at $r = 0$, and is an essentially different mechanism to that of boundary-layer collision above a heated cylinder, for example, as described in ref. [5].

2. EQUATIONS OF MOTION

The equations of motion describing the boundary-layer flow above a heated horizontal disk, in non-dimensional form, are from [1, 2],

$$\frac{\partial}{\partial x}[(1-x)u]+\frac{\partial}{\partial z}[(1-x)w] = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$\frac{\partial p}{\partial z} = \theta \tag{3}$$

$$u\frac{\partial \theta}{\partial x}+w\frac{\partial \theta}{\partial z} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2} \tag{4}$$

with boundary conditions that

$$u = w = 0,\quad \frac{\partial \theta}{\partial z} = -1 \quad \text{on} \quad z = 0,$$
$$u \to 0,\quad \theta \to 0 \quad \text{as} \quad z \to \infty. \tag{5}$$

Here x and z are (non-dimensional) coordinates measuring distance from the edge of the disk and normal to it respectively, with u and w the velocity components in the x- and z-directions respectively; p is the pressure, θ the temperature difference and Pr the Prandtl number. The non-dimensional quantities x, z, u, w, p and θ are related to their dimensional counterparts $\bar{x}, \bar{z}, \bar{u}, \bar{w}, \bar{p}$ and T by $\bar{x} = xa$, $\bar{z} = zaGr^{-1/6}$, $\bar{u} = (\nu/a)Gr^{1/3}u$, $\bar{w} = (\nu/a)Gr^{1/6}w$, $\bar{p} = (\nu^2/a^2)Gr^{2/3}p$ and $T-T_0 = qaGr^{-1/6}\theta$, where $Gr = g\beta qa^4/\nu^2$ is the Grashof number and we are applying the boundary condition that $\partial T/\partial \bar{z} = -q$ on $\bar{z} = 0$.

From equation (1) we can define a stream function ψ such that $u = [1/(1-x)](\partial\psi/\partial z)$ and $w = -[1/(1-x)](\partial\psi/\partial x)$. We then make a transformation of equations (2)–(4) motivated by the fact that for small x the flow will be dominated by the plate solution. This

NOMENCLATURE

- a — radius of the disk
- g — acceleration of gravity
- Gr — Grashof number, $g\beta q a^4/\nu^2$
- p — (non-dimensional) pressure
- p_w — pressure on the disk
- Pr — Prandtl number
- q — prescribed constant thermal gradient
- r — (non-dimensional) coordinate from centre of disk
- T — temperature of the fluid
- T_0 — ambient temperature
- u — (non-dimensional) velocity in the x-direction
- w — (non-dimensional) velocity in the z-direction
- x — (non-dimensional) coordinate along the disk
- z — (non-dimensional) coordinate normal to the disk.

Greek symbols

- β — coefficient of expansion
- δ_2 — momentum thickness
- δ_T — temperature thickness
- θ — (non-dimensional) temperature of the fluid
- θ_w — temperature on the disk
- ρ — density of the fluid
- τ_w — skin friction
- ψ — stream function
- ν — kinematic viscosity.

suggests putting

$$\psi = (1-x)x^{2/3}f(x,\eta), \quad \theta = x^{1/3}h(x,\eta),$$
$$p = x^{2/3}G(x,\eta) \quad \text{where} \quad \eta = z/x^{1/3}. \tag{6}$$

Equations (2)–(4) then become

$$\frac{\partial^3 f}{\partial \eta^3} - \frac{2}{3}G + \frac{\eta}{3}\frac{\partial G}{\partial \eta} - x\frac{\partial G}{\partial x} + \frac{2}{3}f\left(\frac{\partial^2 f}{\partial \eta^2}\right) - \frac{1}{3}\left(\frac{\partial f}{\partial \eta}\right)^2 = \frac{x}{1-x}f\frac{\partial^2 f}{\partial \eta^2} + x\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta \partial x} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}\right) \tag{7}$$

$$\frac{\partial G}{\partial \eta} = h \tag{8}$$

$$\frac{1}{Pr}\frac{\partial^2 h}{\partial \eta^2} + \frac{2}{3}f\frac{\partial h}{\partial \eta} - \frac{1}{3}h\frac{\partial f}{\partial \eta} = \frac{x}{1-x}f\frac{\partial h}{\partial \eta} + x\left(\frac{\partial f}{\partial \eta}\frac{\partial h}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial h}{\partial \eta}\right) \tag{9}$$

together with the boundary conditions that

$$f = \frac{\partial f}{\partial \eta} = 0, \quad \frac{\partial \theta}{\partial \eta} = -1 \quad \text{on} \quad \eta = 0,$$
$$\frac{\partial f}{\partial \eta} \to 0, \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty. \tag{10}$$

A solution of equations (7)–(9), valid for small x, can be obtained expanding f, h and G in ascending powers of x. The procedure is straightforward and the details need not be given here. This results in a hierarchy of equations which have to be solved numerically, and we find, for $Pr = 1$, that

$$\theta_w = x^{1/3}(1.7773 + 0.0628x + 0.0340x^2 + \cdots)$$
$$\tau_w = 1.2870 + 0.3994x + 0.4163x^2 + \cdots \tag{11}$$
$$p_w = -x^{2/3}(2.1639 + 0.2800x + 0.2325x^2 + \cdots)$$

where $\theta_w = \theta(x,0)$ and $p_w = p(x,0)$ are the temperature and pressures respectively on the disk and $\tau_w = (\partial u/\partial z)_{z=0}$ is the skin friction.

3. NUMERICAL SOLUTION

A numerical solution was obtained using essentially the same method as that described in ref. [2]. The details are given in [2] and need not be repeated here. The numerical integration started at $x = 0$ with the plate solution and proceeded towards the centre, with the integration being terminated at $x = 0.981563$ as at this point it was found, because of the very rapid thickening of the boundary layer near the centre, that the outer boundary conditions could not be satisfied to sufficient accuracy. The numerical integration was performed for $Pr = 1$ and the results presented are all for this value of Pr.

Figure 1 shows graphs of θ_w and $\psi(x,\infty)$, the value of ψ at the edge of the boundary layer. This figure clearly shows that both $\psi(x,\infty)$ and θ_w are approaching constant values as $x \to 1$, with a more detailed examination of the numerical results showing that $\psi(x,\infty) \to 1.0763$ and $\theta_w \to 1.817$. Figure 2 shows graphs of τ_w and p_w, and we can see that both τ_w and $-p_w$ increase very rapidly near the centre, consistent with the formation of a thin viscous region next to the disk. The very rapid increase in boundary-layer thickness as $x \to 1$ can clearly be seen in Fig. 3 where two representative boundary-layer thicknesses δ_2 and δ_T are plotted. Here we have defined

$$\delta_2 = \int_0^\infty u^2 \, dz \quad \text{and} \quad \delta_T = \int_0^\infty \theta \, dz.$$

The double structure near the centre can also be seen in Figs. 4 and 5 in which u and θ are plotted against z for

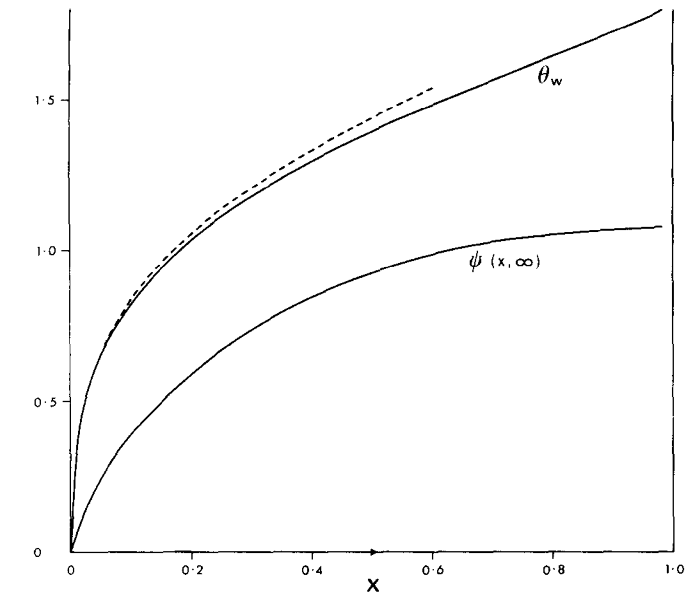

FIG. 1. Graphs of θ_w and $\psi(x, \infty)$ against x (series solution given by broken line).

several values of x. Figure 4 shows that there is a thin region close to the disk where u increases very rapidly to a maximum and then decreases slowly to zero, with the maximum value of u and the spread of this 'tail' also increasing rapidly as x approaches the centre. The same overall pattern is seen in Fig. 5, there being sharp decrease in θ in the thin region next to the disk followed by a long (and rapidly increasing) region in which θ slowly dies away to zero at the edge of the boundary layer.

Also shown in Figs. 1 and 2 are values of τ_w and θ_w as calculated from equation (11). The series solution appears to give a relatively poor estimate for τ_w, being in error by approx. 25% at $x = 0.5$, whereas it seems somewhat better in estimating θ_w, being in error by only 3% at $x = 0.5$.

4. SOLUTION NEAR THE CENTRE

To discuss the solution near the centre of the disk it is more convenient to use $r = 1 - x$, with the centre now at $r = 0$. The numerical solution suggests that near the

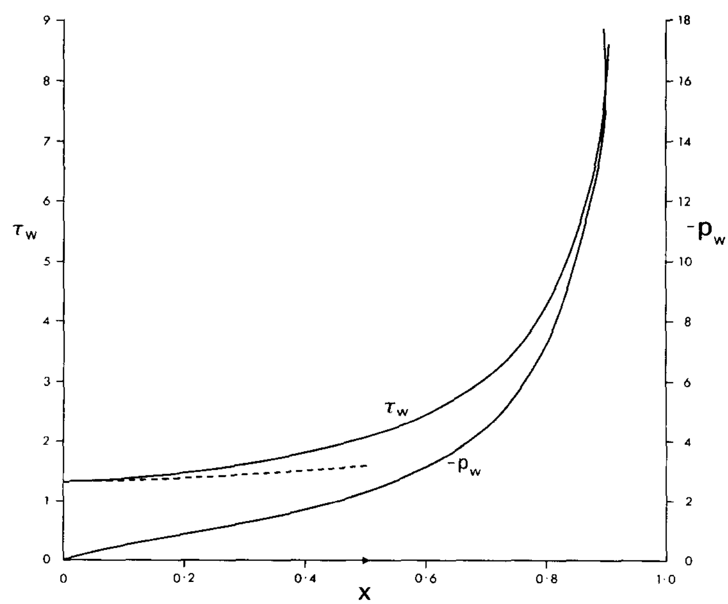

FIG. 2. Graphs of τ_w and $-p_w$ against x (series solution given by broken line).

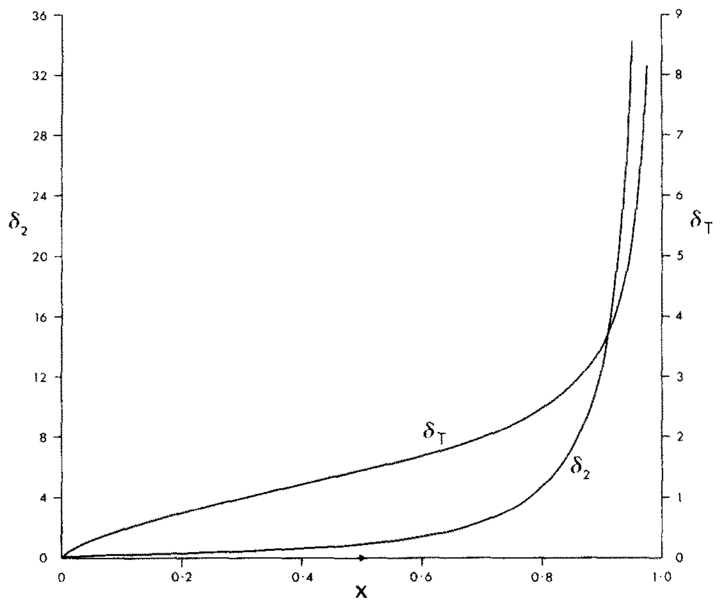

FIG. 3. Graphs of δ_2 and δ_T against x.

centre the pressure p has the form

$$p = -\frac{p_0^2}{2} r^{-m} + p_1 z + \cdots \tag{12}$$

close to the disk, where p_0, p_1 and m are positive constants. The scalings for the inner and outer regions can be determined as follows. Suppose the inner region has thickness $O(r^{\alpha})$, then as a consequence of the boundary condition on θ on $z = 0$, we must have $\theta = p_1 + O(r^{\alpha})$ with $p = -(p_0^2/2)r^{-m} + p_1 z + O(r^{2\alpha})$. A balance between the viscous and convective terms gives ψ of $O(r^{2-\alpha})$. Then a final balance between the pressure gradient and viscous terms gives $\alpha = (2+m)/4$. We now turn to the outer region. Here ψ must be $O(1)$. We have seen this from the numerical solution, but we can also deduce it directly from the equations as this gives the only possibility of obtaining a solution in the outer (inviscid) region in which u and θ die away as $z \to \infty$.

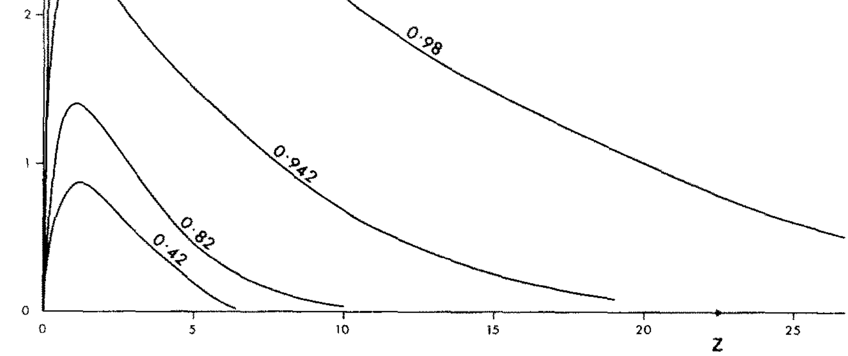

FIG. 4. Graphs of u against z at various x.

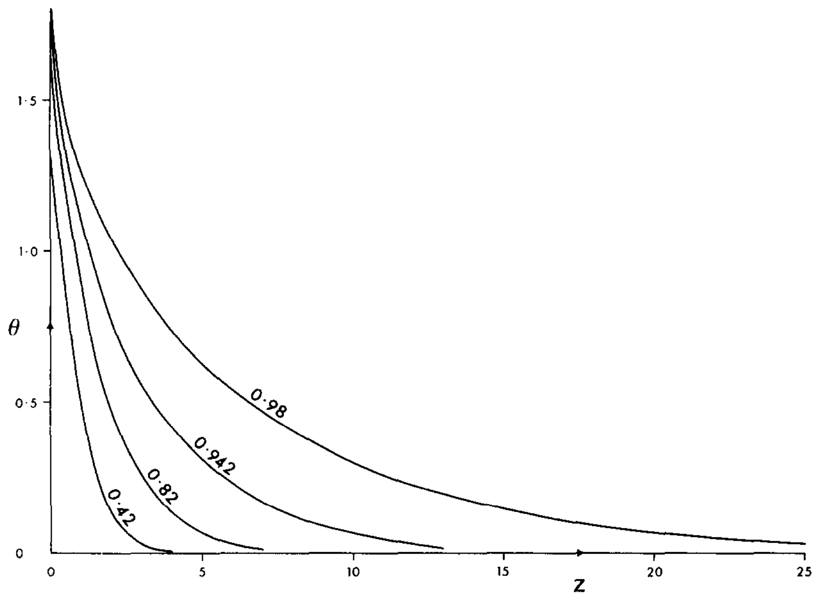

FIG. 5. Graphs of θ against z at various x.

Suppose that the thickness of the outer region is $O(r^{\gamma})$, then [since ψ is $O(1)$] u will be of $O(r^{-1-\gamma})$ but at the outer edge of the inner region the flow must be basically inviscid (to match onto the outer region) with p the same order as u^2, i.e. u will be of $O(r^{-m/2})$, from which it follows that $\gamma = (m-2)/2$. But in the outer region [from equation (3)] $\partial p/\partial z$ must be $O(1)$ [since θ is $O(1)$ to match with the inner region]. Hence it follows that $\gamma = -m$, so that we have $m = \frac{2}{3}$ with $\alpha = \frac{2}{3}$ and $\gamma = -\frac{2}{3}$. It should be noted that here the scaling for the temperature in the inner region is fixed by the matching of the various terms, in contrast to ref. [2] where it emerged from the solution of an eigenvalue problem.

The above discussion suggests that for the inner solution we put

$$\psi = (3p_0)^{1/2} r^{4/3} F(r,\zeta),$$
$$p = -\frac{p_0^2}{2} r^{-2/3} + p_1 z + \frac{3}{p_0} r^{4/3} G(r,\zeta)$$
$$\theta = p_1 + \left(\frac{3}{p_0}\right)^{1/2} r^{2/3} H(r,\zeta)$$

with

$$\zeta = \left(\frac{p_0}{3}\right)^{1/2} \frac{z}{r^{2/3}} \tag{13}$$

where p_0 and p_1 are constants.

The discussion now follows closely that given in ref. [2] (with the important difference that the eigenvalue appearing in [2] is replaced by 4/3). Transformation (13) is substituted into equations (2)–(4) and the resulting equations solved by expanding in powers of r in the form

$$F(r,\zeta) = F_0(\zeta) + \frac{3}{p_0^3} r^2 F_1(\zeta) + \cdots$$
$$G(r,\zeta) = G_0(\zeta) + \frac{3}{p_0^3} r^2 G_1(\zeta) + \cdots \tag{14}$$
$$H(r,\zeta) = H_0(\zeta) + \frac{3}{p_0^3} r^2 H_1(\zeta) + \cdots.$$

The resulting hierarchy of ordinary differential equations are solved subject now only to the inner boundary conditions $F_i(0) = F_i'(0) = 0$ $(i = 0, 1, 2, \ldots)$, $H_0'(0) = -1$, $H_i'(0) = 0$ $(i = 1, 2, \ldots)$ and that the solution should not be exponentially large as $\zeta \to \infty$. The details are similar to those given in [2], with the important feature of their solution being their behaviour as $\zeta \to \infty$. We find that $F_0''(0) = 0.95022$ and that as $\zeta \to \infty$

$$F_0 \sim \zeta - A_0\zeta^{1/2} - \frac{3A_0^2}{8}\log\zeta + \cdots \tag{15}$$

where $A_0 = 2.88$, and that $H_0(0) = -0.2630$ with

$$H_0 \sim B_0\zeta^{1/2}\left(1 - \frac{A_0}{2}\zeta^{-1/2} + \cdots\right) \tag{16}$$

as $\zeta \to \infty$, with $B_0 = -1.875$. Also

$$G_0 \sim C_0 + \frac{2}{3}B_0\zeta^{3/2}\left(1 - \frac{3}{4}A_0\zeta^{-1/2} + \cdots\right) \tag{17}$$

where C_0 is some constant which cannot be determined from this asymptotic expansion.

For the terms of $O(r^2)$ we find that, as $\zeta \to \infty$,

$$F_1 \sim \frac{2}{15} B_0 \zeta^{5/2} + A_1 \zeta^2 + \cdots - C_0\left(\zeta + \frac{3}{8} A_0 \zeta^{1/2} + \cdots\right) \tag{18}$$

$$H_1 \sim B_1 \zeta^2 + \tfrac{1}{2}\left(B_0 A_1 + B_0^2 \frac{A_0}{3} - 4A_0 B_0\right)\zeta^{3/2} + \cdots - \frac{C_0 B_0}{2}\zeta^{1/2} + \cdots \tag{19}$$

$$G_1 \sim C_1 + B_1 \frac{\zeta^3}{3} + \frac{1}{5}\left(B_0 A_1 + B_0^2 \frac{A_0}{3} - 4A_0 B_1\right)\zeta^{5/2} + \cdots - \frac{C_0 B_0}{3}\zeta^{3/2} + \cdots \tag{20}$$

where A_1, B_1 and C_1 are constants.

We are now in a position to consider the outer region. The argument given at the beginning of this section suggests we put

$$\psi = \frac{p_0^3}{2}\phi(r,y), \quad p = \frac{p_0^2}{2} r^{-2/3} g(r,y)$$
$$\theta = \theta(r,y) \quad \text{where} \quad y = \frac{2}{p_0^2} r^{2/3} z. \tag{21}$$

The boundary conditions to be satisfied are that $\partial\phi/\partial y \to 0$, $\theta \to 0$, $g \to 0$ as $y \to \infty$, and from equations (15)–(20) that, for small y,

$$\phi = y + \frac{\bar{B}_0}{15} y^{5/2} + \cdots + r^{2/3}\left(-\bar{A}_0 y^{1/2} + \frac{\bar{A}_1}{2p_0} y^2 + \cdots\right) + \cdots \tag{22}$$

$$\theta = p_1 + \frac{\bar{B}_0}{p_0} y^{1/2} + \frac{3\bar{B}_1}{2p_0} y^2 + \cdots + r^{2/3}\left[-\frac{\bar{A}_0 \bar{B}_0}{2p_0} + \left(\bar{A}_1 \bar{B}_0 + \bar{A}_0 \frac{\bar{B}_0^2}{3} - 12 p_0 \bar{A}_0 \bar{B}_1\right)\frac{y^{3/2}}{2p_0^2} + \cdots\right] \tag{23}$$

$$g = -1 + p_1 y + \frac{2}{3}\frac{\bar{B}_0}{p_0} y^{3/2} + \frac{\bar{B}_1}{2p_0} y^3 + \cdots + r^{2/3}\left[-\frac{\bar{A}_0 \bar{B}_0}{2p_0} y + \left(\bar{A}_1 \bar{B}_0 + \frac{\bar{A}_0 \bar{B}_0^2}{3} - 12 p_0 \bar{A}_0 \bar{B}_1\right)\frac{y^{5/2}}{5p_0^2} + \cdots\right] + \cdots \tag{24}$$

where $\bar{A}_0 = A_0 k^{-1/2}$, $\bar{B}_0 = (3p_0)^{1/2} k^{1/2} \bar{B}_0$, $\bar{A}_1 = (3p_0)^{1/2} \bar{A}_1$ and $\bar{B}_1 = k\bar{B}_1$, and $k = (p_0^5/12)^{1/2}$.

The equations obtained by substituting transformation (21) into equations (2)–(4) are the same as those given in ref. [2] for the outer region with the form of equations (22)–(24) suggesting expansions for ϕ, g and θ as

$$\phi = \phi_0(y) + r^{2/3}\phi_1(y) + \cdots$$
$$\theta = \theta_0(y) + r^{2/3}\theta_1(y) + \cdots \tag{25}$$
$$g = g_0(y) + r^{2/3} g_1(y) + \cdots.$$

The leading order terms satisfy

$$\phi_0'^2 + g_0 - y g_0' = 0, \quad g_0' = \theta_0 \tag{26}$$

(now primes denote differentiation with respect to y). Equations (26) are equivalent to Bernoulli's equation and show that one of the functions $g_0(y)$, say, is arbitrary, the others being determined in terms of this one. We know only that $g_0 \to 0$ as $y \to \infty$ and that for small y, from (24),

$$g_0 = -1 + p_1 y + \frac{2}{3}\frac{\bar{B}_0}{p_0} y^{3/2} + \cdots. \tag{27}$$

We can also check that the forms for ϕ_0 and θ_0 for small y as implied by equations (22) and (23) are consistent with (26).

The equations for the terms of $O(r^{2/3})$ are

$$2\phi_0''\phi_1 - y g_1' = 0, \quad \theta_1 = g_1', \quad \phi_0'\theta_1 - \phi_1\theta_0' = 0. \tag{28}$$

Again these equations cannot be solved explicitly, for if we eliminate θ_1 we get $2\phi_0''\phi_1 = y\phi_1(\theta_0'/\phi_0')$, from which ϕ_1 can be cancelled to give $2\phi_0'\phi_0'' = y\theta_0'$. This is just equation (26) differentiated once with respect to y.

Finally it remains to consider the behaviour of the numerical solution as $r \to 0$. Values of $\psi(r, \infty)$ and θ_w are given in Table 1. Clearly $\psi(r,\infty)$ is approaching a constant value, of 1.0763. The evidence for θ_w is less clear, but θ_w does appear to be tending to a constant, of approx. 1.817 as $r \to 0$. Transformation (13) implies that the skin friction τ_w should be $O(r^{-1})$ for small r. This behaviour can be seen in Fig. 2 and more directly in Table 1 where values of $\tau_w^* = r\tau_w$ are given. These results suggest that τ_w^* is in fact tending to a constant value of approx. 0.678, as $r \to 0$. We can also check the boundary-layer thicknesses δ_2 and δ_T. We know from equation (21) that δ_2 and δ_T should be $O(r^{-4/3})$ and

Table 1. Values of $\psi(r,\infty)$, θ_w, τ_w^*, δ_2^* and δ_T^* for various r

r	$\psi(r,\infty)$	θ_w	τ_w^*	δ_2^*	δ_T^*
0.20	1.0535	1.643	0.922	0.550	0.838
0.16	1.0606	1.674	0.814	0.569	0.809
0.12	1.0668	1.705	0.783	0.588	0.778
0.08	1.0745	1.739	0.777	0.607	0.743
0.07	1.0750	1.747	0.746	0.611	0.733
0.06	1.0754	1.756	0.737	0.615	0.723
0.05	1.0757	1.765	0.729	0.619	0.712
0.045	1.0759	1.769	0.725	0.621	0.707
0.04	1.0760	1.774	0.720	0.623	0.702
0.035	1.0761	1.779	0.715	0.625	0.697
0.03	1.0762	1.784	0.711	0.626	0.692
0.025	1.0763	1.790	0.706	0.628	0.686
0.02	1.0763	1.795	0.700	0.630	0.682

$O(r^{-2/3})$ respectively for small r. Values of $\delta_2^* = r^{4/3}\delta_2$ and $\delta_T^* = r^{2/3}\delta_T$ are also given in Table 1. These again show that δ_2^* and δ_T^* are both approaching constant values (of approx. 0.637 and 0.662 respectively) as $r \to 0$. From equation (3) we have that $p_w = -\delta_T$, and from the limit of δ_T^* as $r \to 0$ we calculate p_0 as 1.151, which in turn implies, from (21) and the given value of $F_0''(0)$, that $\tau_w^* \to 0.677$.

The results given in Table 1 give a convincing confirmation of the theory presented in this section, bearing in mind that no particular form was forced on the numerical solution near the centre and consequently, for small r, the inner region is very thin in terms of η.

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CONVECTION NATURELLE AU DESSUS D'UN DISQUE CIRCULAIRE HORIZONTAL ET UNIFORMEMENT CHAUFFE

Résumé—On considère la couche limite de convection naturelle au dessus d'un disque horizontal et chauffé uniformément. Les équations du mouvement sont résolues numériquement et par un développement en série valable près de la circonférence. On montre que près du bord du disque, l'écoulement est le même que sur une plaque, avec une importance croissante des effets de courbure quand on s'approche du centre. On montre aussi que, près du centre, l'épaisseur de la couche limite augmente très rapidement et que la solution se partage en deux régions distinctes. Il y a une mince région visqueuse près du disque d'épaisseur $O(r^{2/3})$, où r mesure la distance du centre et pour une plus épaisse couche non visqueuse d'épaisseur $O(r^{-2/3})$.

FREIE KONVEKTION AN EINER GLEICHFÖRMIG BEHEIZTEN, HORIZONTALEN KREISSCHEIBE

Zusammenfassung—Es wird die Grenzschicht bei freier Konvektion über einer gleichförmig beheizten, horizontalen Scheibe untersucht. Die Bewegungsgleichungen werden numerisch und außerdem durch eine in der Berandungsnähe gültige Reihenentwicklung gelöst. Es wird gezeigt, daß in der Nähe der Scheibenberandung die grundlegend gleiche Strömung wie auf einer Platte vorliegt, wobei die Krümmungseinflüsse bei Annäherung an die Scheibenmitte an Bedeutung gewinnen. Ebseno wird gezeigt, daß in der Nähe der Scheibenmitte die Grenzschichtdicke sehr stark ansteigt und die Lösung sich in zwei getrennte Gebiete aufspaltet. Es existiert ein dünnes viskoses Gebiet nahe an der Scheibe mit einer Dicke $0(r^{2/3})$, wobei r der Abstand von der Scheibenmitte ist, und ein sehr viel dickeres nicht-viskoses Gebiet von der Dicke $0(r^{-2/3})$.

СВОБОДНАЯ КОНВЕКЦИЯ НАД РАВНОМЕРНО НАГРЕТЫМ ГОРИЗОНТАДЬНЫМ КРУГЛЫМ ДИСКОМ

Аннотация—Рассматривается свободноконвективный пограничный слой над равномерно нагретым горизонтальным диском. Уравнения движения решаются численно и с помощью разложения в ряд, справедливого вблизи кромки диска. Показано, что у края диска течение, в основном, имеет такой же характер как и на пластине, а влияние кривизны возрастает по мере приближения к центру. Показано также, что вблизи центра толщина пограничного слоя возрастает очень быстро и решение расщепляется на две отчетливых области. Около диска имеется тонкая вязкая область толщиной $O(r^{2/3})$, где r расстояние от центра, и значительно более толстая невязкая область толщиной $O(r^{-2/3})$.